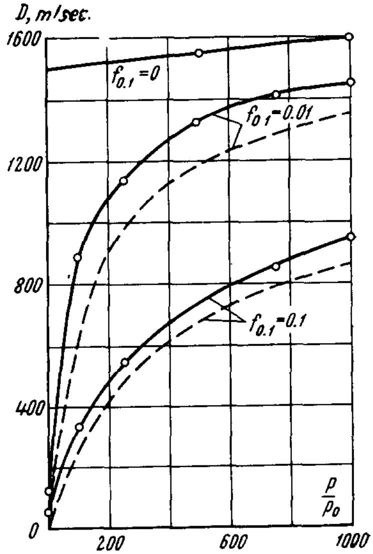

Fig. 1

$$\psi_j = \rho_{j0}^{\circ}\left[1 + \frac{k_j(p-p_0)}{\rho_{j0}^{\circ}c_{j0}^2}\right]^{1/k_j} \qquad (j=2,3) \qquad \text{(cont.)}$$

$$\varkappa = (\gamma_1+1)/(\gamma_1-1)$$

Here the constants k_2 and k_3 denote the isentropic indices for the liquid and solid component and γ_1 is the gas isentropic exponent (the gas is assumed perfect).

In particular for air, water and quartz under the normal conditions we can assume that the true mass densities are 0.125, 102 and 265 kg$\cdot$sec^2/m^4, the velocities of sound are 330, 1500 and 4500 m/sec and the isentropic exponents are 1.4, 3 and 3, respectively. By virtue of the above, for air and water we have, respectively, $\rho^{\circ}_{10}c_{10}^2 = 1.31\cdot10^4$ kg/m^2 and $\rho_{20}^{\circ}c_{10}^2 = 2.25\cdot10^8$ kg/m^2. Using these values together with the formula (1.10) we find, that, for $f_1 = 0.01$ the velocity of sound in the air-water mixtute is $c = 114$ m/sec, while for $f_1 = 0.1$ it falls to 38 m/sec which is less than the velocity of sound in any of the components. The Fig. 1 depicts the velocities of the shock waves for air-water mixture obtained from (2.4) and (2.5), plotted (in solid lines) as the functions of f_{10} versus the pressure ratio p/p_0 at the discontinuity where $p_0 = 1$ atm. Results of [2] are shown in broken lines.

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Translated by L. K.

USE OF A VARIATIONAL PRINCIPLE FOR THE STUDY OF PROPAGATION OF SURFACES OF DISCONTINUITY IN A CONTINUOUS MEDIUM

PMM Vol. 33, №4, 1969, pp. 602-608
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(Received January 1969)

Variational principles were used as a starting point for constructing models of various continuous media in [1−5], where their application was studied in detail. In the present paper which is a continuation of [6], the generalized variational relation is extended to embrace the media possessing surfaces of discontinuity of the crack type. A problem concerning the character of a singular solution to the plane problem near the contour of a spreading crack is considered for a medium whose energy and stresses depend on the gradient of the deformation tensor.

1. Let us consider an arbitrary isolated volume V (ξ^1, ξ^2, ξ^3, t) of a continuous medium referred to the Lagrangian coordinates ξ^1, ξ^2 and ξ^3 and the time t and containing a part of the discontinuity surface Σ (Fig. 1).

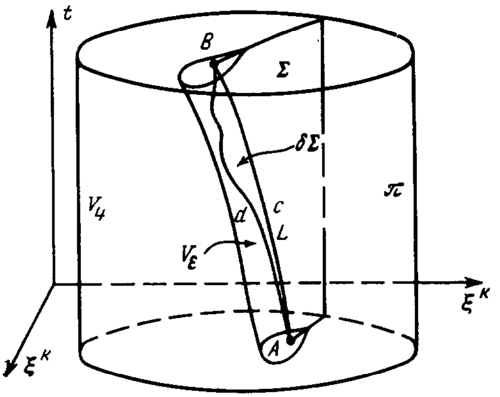

Fig. 1

The law of motion represented by the functions

$$x^i = x^i(\xi^1, \xi^2, \xi^3, t) =$$
$$= \xi^i + u^i(\xi^1, \xi^2, \xi^3, t)$$
$$x^4 = t \qquad (i = 1, 2, 3)$$

will constitute the required relation in the observer (*) system defining the motion of the medium.

In the Newtonian mechanics we assume that the equality $x^4 = t$ holds, and the absolute time is considered as a scalar quantity.

In paper [6] we considered the surfaces of discontinuity which could move freely over the particles of the medium. The quantity δl_n determining the normal displacement of the surface, was different from zero. In this paper we consider the case when $\delta l_n = 0$ and where all variations $\delta\Sigma$ of the surface of discontinuity result from the fact that it is imbedded in the medium along the hyperbolic arc A_cB. The slope of this arc characterizes the rate of such an imbedding, i.e. the rate of propagation of the contour of a two-dimensional surface of discontinuity past the particles of the medium, in the space of initial states. We shall assume, for definiteness, that the particle displacement vector becomes discontinuous at the surface Σ. Obviously, Σ will be cylindrical with respect to time within the space in question.

We shall use the following variational relation [1]:

$$\delta \int_{V_4} \Lambda \sqrt{g}\, d\tau_4 + \delta W + \delta W^* = 0 \qquad (1.1)$$

to obtain the conditions at the surface Σ.

Here Λ is a Lagrangian function depending, in a manner admitted by the invariance considerations, on the particle velocity v^ω, the initial density ρ_0, the entropy S, the metric tensors g_{ik}° and g_{ik} and on the gradients $\nabla_k^\circ g_{ij}$ $(g = \det\| g_{ik} \|)$. The functional δW^* which is given, contains terms describing the entropy changes, the body forces K_ω, and the energy necessary to form a new area element of the surface of discontinuity

*) We shall construct our discussion within the framework of the theory of finite deformations and for this reason we must keep in mind two additional systems apart from the observer's system, namely the actual system defining the metric g_{ik} (ξ^α, t) of the present instant and the initial system with a metric tensor g_{ik}° $(\xi^\alpha) = g_{ik}(\xi^\alpha, t_0)$. The latter and the observer's system, can both be assumed Cartesian, provided that $g_{ik}^\circ = \delta_{ik}$.

$$\delta W^* = \int_{V_4} (\rho T \delta S + \rho K_\omega \delta u^\omega + \rho K_\omega v^\omega \delta x^4)\, d\tau_4 + \int_{A_cB} 2\gamma \delta \Sigma_3 \tag{1.2}$$

Functional δW is represented by an integral of the linear combination of the defining parameters taken over the boundary π of the four-dimensional volume V_4 and can be expressed in terms of the known Λ and δW^*[1].

In evaluating the variations of action in (1.1) we take into account the fact that not only the parameters of the medium, but also the surface of discontinuity Σ along A_cB itself will be the subject to variation. Let the surface $\Sigma + \delta\Sigma$ (Fig. 1) act as reference for the variable surface of discontinuity. The variation of action is defined as the principal linear part of the difference of integrals computed over the volume $V_4'' = V_4 - (\Sigma + \delta\Sigma)$ and the volume $V_4' = V_4 - \Sigma$

$$\delta \int_{V_4} \Lambda \sqrt{g}\, d\tau_4 = \int_{V_4''} (\Lambda \sqrt{g})\, d\tau_4 - \int_{V_4'} \Lambda \sqrt{g}\, d\tau_4 \tag{1.3}$$

To compute this difference we shall change the region of integration of the first integral to $V_4 - \Sigma$, using the following change of variable

$$t' = t + \delta t\,(\xi^\alpha, t), \qquad \delta t\,|_\pi = 0$$

the latter transforming $A_c B$ into $A_d B$. Assuming that the Jacobian of the transformation is equal to $1 + \partial\delta t / \partial t$ with the accuracy of up to the higher order infinitesimals, we obtain

$$\delta \int_{V_4} \Lambda \sqrt{g}\, d\tau_4 = \int_{V_4'} \delta(\Lambda \sqrt{g})\, d\tau_4 + \int_{V_4'} \Lambda \sqrt{g}\, \frac{\partial \delta t}{\partial t}\, d\tau_4 \tag{1.4}$$

All variations appearing in the first term of (1.4) are computed in the manner analogous to that used in [6]. At the same time we assume that the total variation δ_1 of all quantities is given by

$$\delta_1 A = (\delta A + A^{\cdot}\delta t) + A^{\cdot}\delta x^4 \tag{1.5}$$

The variation δ_1 is understood to be a variation of the integral appearing in the left part of (1.4). The variation δx^4 is assumed an arbitrary constant in Newtonian mechanics.

Assuming now that the function $\Lambda\sqrt{g}$ undergoing the variation may have integrable singularities on the contour A_cB, we shall compute the integrals in (1.4) over the region $V_4 - V_\varepsilon - \Sigma$ where V_ε denotes a narrow ε-tube enveloping the contour A_cB and merging smoothly with the surface Σ. Having performed the variation we find, that the volume integral contains the variational Lagrange's equations. Moreover, the equations of motion will accompany the variations δu^ω, the energy equations the variation δx^4 and the thermodynamic relation the variation δS. We also obtain the boundary conditions on the main part of the surface Σ and assume them to hold (in the present case these conditions reduce to the absence of the force and momentum stresses on Σ). Assuming further that all variations on the surface π are equal to zero we obtain, after the usual manipulations (*),

$$\lim_{\varepsilon \to 0} \left[\int_{\Sigma_\varepsilon} \left(J_\omega n_t - \sqrt{g}\, \frac{\partial \xi^k}{\partial x^\omega}\, p_k{}^j n_j{}^\circ \right) \delta u^\omega d\sigma_3 + \int_{\Sigma_\varepsilon} (\Lambda \sqrt{g} n_t \delta t + \right.$$

*) Certain inaccuracies which occurred in transforming the surface integral in [6], were brought to the author's attention by Zhelnorovich [3]. For this reason the following manipulations are performed independently of [6].

$$Q^{kij} n_k{}^\circ \delta g_{ij})\, d\sigma_3 + \int\limits_{A_c B} 2\gamma n_t \delta t\, d\lambda_2 = 0 \qquad (1.6)$$

Here Σ_ε denotes the surface of the four-dimensional volume V_ε, $d\sigma_3$ is the area element of this surface, $d\lambda_2$ is the element of the hyperarc A_cB, n_t and $n_j{}^\circ$ denote the components of the vector normal to the surface Σ_ε and to the contour A_cB in the adjacent space and the quantities J_ω, $p_k{}^j$ and Q^{kij} are defined by [6]

$$J_\omega = \frac{\partial \Lambda \sqrt{g}}{\partial v^\omega}, \qquad Q^{kij} = \frac{\partial \Lambda \sqrt{g}}{\partial \nabla_k{}^\circ g_{ij}}$$
$$p^{ij} = -\frac{2}{\sqrt{g}} \frac{\partial \Lambda \sqrt{g}}{\partial g_{ij}} + 2\sqrt{\frac{g_0}{g}}\, \nabla_k{}^\circ \frac{1}{\sqrt{g_0}} \frac{\partial \Lambda \sqrt{g}}{\nabla_k{}^\circ g_{ij}} \qquad (1.7)$$

The integration over the space-time surface Σ_ε and the space-time arc A_cB in (1.6) can be performed by integration over their spatial parts Σ_ε ($d\sigma_2$) and L ($d\lambda_1$), and the time t. The spatial parts are obtained from Σ_ε and A_cB by taking a cross section $t = \mathrm{const}$. This can be done according to the following formulas:

$$|\, \overset{\circ}{n}_j \,|\, d\sigma_3 = d\sigma_2 dt, \qquad |\, \overset{\circ}{n}_j \,|\, d\lambda_2 = d\lambda_1 dt$$

Let us introduce the notation $N = -n_t \,/\, |\overset{\circ}{n}_j|$ and $n_j = \overset{\circ}{n}_j \,/\, |\overset{\circ}{n}_j|$. Then, omitting the integration with respect to time, we obtain

$$\lim_{\varepsilon\to 0} \Big[\int\limits_{\Sigma_\varepsilon} \Big(J_\omega N + \sqrt{g}\, \frac{\partial \xi^k}{\partial x^\omega} p_k{}^j n_j \Big) \delta u^\omega\, d\sigma_2 +$$
$$+ \int\limits_{\Sigma_\varepsilon} (\Lambda \sqrt{g} N \delta t - Q^{kij} n_k \delta g_{ij})\, d\sigma_2 + \int\limits_L 2\gamma N \delta t\, d\lambda_1 \Big] = 0 \qquad (1.8)$$

To perform further transformations, we must isolate the independent parts of the variations $\delta g_{ij} = \nabla_i{}^\wedge \delta u_j{}^\wedge + \nabla_j{}^\wedge \delta u_i{}^\wedge$ defined on the surface. These will be the variations of displacements and their derivatives in the normal direction. Using the relations

$$(1)\;\; Q^{kij} \delta g_{ij} = 2Q^{kij} \nabla_i{}^\wedge \Big(g_{js} \frac{\partial \xi^s}{\partial x^\omega} \delta u^\omega \Big) =$$
$$= 2Q_s^{ki} \Big[\frac{\partial \xi^p}{\partial x^\omega} \Gamma_{ip}^s - \frac{\partial \xi^s}{\partial x^p} \frac{\partial}{\partial x^\omega} \Big(\frac{\partial x^p}{\partial \xi^i} \Big) \Big] \delta u^\omega + 2Q_s^{ki} \frac{\partial \xi^s}{\partial x^\omega} \frac{\partial \delta u^\omega}{\partial \xi^i}$$
$$(2)\;\; \frac{\partial}{\partial \xi^i} = (\delta_i{}^\alpha - n_i n^\alpha) \frac{\partial}{\partial \xi^\alpha} + n_i n^\alpha \frac{\partial}{\partial \xi^\alpha} = D_i + n_i \frac{\partial}{\partial n}$$

we can write (1.8) in the form

$$\lim_{\varepsilon\to 0} \Big\{ \int\limits_{\Sigma_\varepsilon} \Big(J_\omega N + \sqrt{g}\, \frac{\partial \xi^k}{\partial x^\omega} p_k{}^j n_j + 2\Omega_{i\omega}^s Q_s^{ki} n_k \Big) \delta_1 u^\omega\, d\sigma_2 -$$
$$- 2 \int\limits_{\Sigma_\varepsilon} \frac{\partial \xi^s}{\partial x^\omega} Q_s^{ki} n_i n_k \delta_1 \frac{\partial u^\omega}{\partial n}\, d\sigma_2 +$$
$$+ \int\limits_{\Sigma_\varepsilon} \Big[(\Lambda \sqrt{g} - J_\omega v^\omega) N - \Big(\sqrt{g}\, \frac{\partial \xi^k}{\partial x^\omega} p_k{}^j n_j + 2\Omega_{i\omega}^s Q_s^{ki} n_k \Big) v^\omega +$$
$$+ 2 \frac{\partial \xi^s}{\partial x^\omega} Q_s^{ki} n_i n_k \frac{\partial v^\omega}{\partial n} \Big] \delta t\, d\sigma_2 \Big\} + \int\limits_L 2\gamma N \delta t\, d\lambda_1 = 0 \qquad (1.9)$$

$$\Omega_{i\omega}^s = (n_i D_\alpha n^\alpha - D_i) \frac{\partial \xi^s}{\partial x^\omega} + \frac{\partial \xi^p}{\partial x^\omega} \Gamma_{ip}^s - \frac{\partial \xi^s}{\partial x^p} \frac{\partial}{\partial x^\omega} \Big(\frac{\partial x^p}{\partial \xi^i} \Big)$$

Taking into account the arbitrariness of the variations entering (1. 9), we obtain the required relations which hold along the edge of the crack. These relations can be written more conveniently by introducing the cross section perpendicular to the contour L of the crack, using the line of intersection Γ_ε of this cross section with the surface Σ_ε as a directrix, and regarding the surface Σ_ε itself as formed by the lines parallel to the contour L and passing through Γ_ε. We then obtain

$$\lim_{\varepsilon\to 0}\int_{\Gamma_\varepsilon}\left(J_\omega N+\sqrt{g}\,\frac{\partial\xi^k}{\partial x^\omega}\,p_k{}^j n_j+2\Omega^s_{i\omega}Q_s^{ki}n_k\right)d\lambda_1=0 \tag{1.10}$$

$$\lim_{\varepsilon\to 0}\int_{\Gamma_\varepsilon}\frac{\partial\xi^s}{\partial x^\omega}\,Q_s^{ki}n_i n_k\,d\lambda_1=0 \tag{1.11}$$

$$\lim_{\varepsilon\to 0}\int_{\Gamma_\varepsilon}\Big[(J_\omega v^\omega-\Lambda\sqrt{g})\,N+\left(\sqrt{g}\,\frac{\partial\xi^k}{\partial x^\omega}\,p_k{}^j n_j+2\Omega^s_{i\omega}Q_s^{ki}n_k\right)v^\omega-$$
$$-2\,\frac{\partial\xi^s}{\partial x^\omega}\,Q_s^{ki}n_i n_k\frac{\partial v^\omega}{\partial n}\Big]\,d\lambda_1=2\gamma N \tag{1.12}$$

Here $d\lambda_1$ is an element of the contour Γ_ε. Obviously, N is the rate of motion of the contour L through the space of initial states, while in the contour integral it denotes the projection of this velocity on the direction normal to Σ_ε.

Relations (1. 10) and (1. 11) reflect the fact that no external concentrated forces which could be described by the term δW^*, act on the contour of the surface of discontinuity.

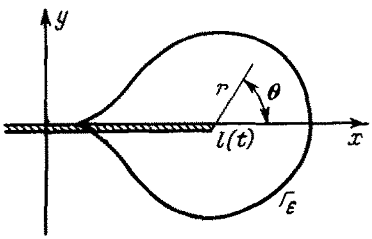

Fig. 2

Relation (1. 12) will constitute an additional independent condition representing the energy balance at the edge of Σ_ε. When the deformations are small, i. e. when
$$\varepsilon_{ij}=\frac{1}{2}(g_{ij}-g_{ij}{}^\circ)\approx\frac{1}{2}\left(\frac{\partial u_i}{\partial\xi^j}+\frac{\partial u_j}{\partial\xi^i}\right)$$
in which case we express the Lagrangian function by $L={}^1/_2\rho\,v^2-\rho\,U\,(\varepsilon_{ij})$, where U is the energy of a unit mass, the expression (1. 12) simplifies considerably coinciding with the analogous equation (1. 1) of [7].

2. We shall consider the problem on propagation of the surface of discontinuity of the crack type in a medium in which the energy and stresses depend on the deformations and their gradients. A system of defining equations can be obtained for such a medium from the variational relations (1. 1) in which the expression $L={}^1/_2\rho\,v^2-\rho\,U(\varepsilon_{ij},\nabla_k\varepsilon_{ij})$ is taken as the Lagrangian function and U is a positive definite quadratic function of its arguments
$$U=\frac{\lambda}{2}\varepsilon_{ii}{}^2+\mu\varepsilon_{ij}\varepsilon_{ij}+k_1\varepsilon_{ij,k}\varepsilon_{ij,k}+k_2\varepsilon_{ij,k}\varepsilon_{ik,j}+$$
$$+k_3\varepsilon_{ij,i}\varepsilon_{kj,k}+k_4\varepsilon_{ij,i}\varepsilon_{kk,j}+k_5\varepsilon_{ii,j}\varepsilon_{kk,j} \tag{2.1}$$

Here λ, μ, k_1, k_2, k_3, k_4 and k_5 are the elastic constants. In particular, the stresses in such a medium are given by
$$\frac{1}{\rho_0}\,p_{ij}=\frac{\partial U}{\partial\varepsilon_{ij}}-\frac{\partial Q_{kij}}{\partial x_k},\qquad Q_{kij}=\frac{\partial U}{\partial\nabla_k\varepsilon_{ij}} \tag{2.2}$$

Assuming that the state of plane deformation prevails in the continuous medium in the small neighborhood of each point of the crack contour, we shall direct the Z-axis along the surface of the crack, the X-axis in the direction of propagation of the crack and the Y-axis in the direction perpendicular to it (Fig. 2). The function $l\ (t)$ will define the law of motion of the crack edge. Basic relations of the problem follow [8]. The equations of motion are

$$\rho_0 \frac{\partial^2 u_i}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial x^i} + \mu \nabla^2 u_i - \nabla^2 \left(A \frac{\partial \Delta}{\partial x^i} + B \nabla^2 u_i \right) \qquad (i = 1, 2) \tag{2.3}$$

and the stresses expressed in terms of displacements are

$$\frac{1}{\rho_0} p_{\alpha\alpha} = \lambda \Delta + 2\mu \frac{\partial u_\alpha}{\partial x_\alpha} - \left[2\tau_1 \nabla^2 \frac{\partial u_\alpha}{\partial x_\alpha} + \tau_2 \frac{\partial^2 \Delta}{\partial x_\alpha^2} + \tau_3 \nabla^2 \Delta \right] \tag{2.4}$$

$$\frac{1}{\rho_0} p_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - \left[\tau_1 \nabla^2 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \tau_2 \frac{\partial^2 \Delta}{\partial x \partial y} \right] \qquad \Delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

Here λ, μ, τ_1, τ_2, τ_3, A and B are elastic constants, last five of which are linear functions of k_1, k_2, k_3, k_4 and k_5.

Denoting now the polar components of the displacement vector by u_r and u_θ, we shall seek a singular solution of the problem near the crack contour, in the following form [9]

$$\begin{aligned} u_r &= u_{r0}\ (t) + r^m \alpha\ (\theta,\ t) + o\ (r^m) \\ u_\theta &= u_{\theta 0}\ (t) + r^m \beta\ (\theta,\ t) + o\ (r^m) \end{aligned} \tag{2.5}$$

where $\alpha(\theta,\ t)$ and $\beta\ (\theta,\ t)$ are unknown functions, $m > 0$ is an undefined constant, $r = [(x - l)^2 + y^2]^{1/2}$ is the distance to the crack edge and θ is the polar angle.

The required value of the exponent m defining the character of the singularity can be obtained, as in other similar cases [7], without solving the boundary value problem for (2.3). Condition (1.12) yields

$$\lim_{\varepsilon \to 0} \int_{\Gamma_\varepsilon} \left\{ \left(\frac{1}{2} \rho_0 v^2 + \rho_0 U \right) l^{\cdot} n_1 - \left[p_{i\omega} n_i + 2\Omega_{i\omega s} \left(\rho_0 \frac{\partial U}{\partial \nabla_k \varepsilon_{is}} n_k \right) \right] v_\omega + \right.$$
$$\left. + 2\rho_0 \frac{\partial U}{\partial \nabla_k \varepsilon_{i\omega}} n_i n_k \frac{\partial v_\omega}{\partial n} \right\} d\lambda_1 = 2\gamma l^{\cdot} \tag{2.6}$$

where $v_\omega = \partial u_\omega / \partial t$ denotes the particle velocity, Γ_ε is an arbitrary contour surrounding the end of the crack $l\ (t)$, $n_1 = \cos(nx)$, $n_2 = \cos\ (ny)$ and $l^{\cdot}$ is the rate of propagation of the crack. All functions appearing in (2.6) have been taken directly from the singular solution and we see from (2.6) that the integrand function sould exhibit a singularity of the type r^{-1} when $r \to 0$. Taking into account the fact that

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial t}\bigg|_{l = \text{const}} - \frac{\partial u}{\partial x} l^{\cdot}$$

we easily see that the highest terms appearing in the expression under the integral sign are of the order $2m - 4$. From this, the necessary condition $2m - 4 = -1$ of the energy balance yields $m = {}^3/_2$.

Simple manipulations confirm that solution (2.5) satisfies also the corresponding boundary value problem for Eqs. (2.3) with the accuracy of up to the higher order infinitesimals. Assuming that u_r is an even and u_θ is an odd function of θ and inserting these functions into (2.3), we obtain the following expressions for $\alpha(\theta,\ t)$ and $\beta\ (\theta,\ t)$:

$$\begin{aligned} \alpha(\theta,\ t) &= c_1 \cos\ (m + 1)\ \theta + c_2 \cos\ (m - 1)\ \theta + c_3 \cos\ (m - 3)\theta \\ \beta(\theta,\ t) &= -c_1 \sin\ (m + 1)\theta + c_4 \sin\ (m - 1)\ \theta + \sigma c_3 \sin\ (m - 3)\ \theta \end{aligned} \tag{2.7}$$

where $c_i\ (t)$ are arbitrary functions of time and σ is a known constant.

Assuming now that the surface of the crack is load-free, we obtain from the system of boundary conditions the following four homogeneous equations [2] for the four functions of time $c_i\ (t)$

$$p_{xy} - 2\frac{\partial}{\partial x}\left(\rho_0 \frac{\partial U}{\partial \varepsilon_{xx,y}}\right) = 0, \quad \theta = \pi$$

$$p_{yy} - 2\frac{\partial}{\partial x}\left(\rho_0 \frac{\partial U}{\partial \varepsilon_{xy,y}}\right) = 0, \quad \theta = \pi \tag{2.8}$$

$$\frac{\partial U}{\partial \varepsilon_{xy,y}} = \frac{\partial U}{\partial \varepsilon_{yy,y}} = 0, \quad \theta = \pi$$

When the function U is independent of the deformation gradients, these conditions coincide with the usual ones. Inserting (2.7) into (2.8) and computing the resulting determinant for the system of four equations for c_i we find, that the determinant vanishes when $m = {}^3/_2$ (due to the particular form of (2.7), two rows of the determinant become equal to zero when $m = {}^3/_2$ and $\theta = \pi$) thus yielding a nontrivial solution to the problem (2.3)–(2.8). Using functions (2.7) we can follow the asymptotic distribution of stresses near the crack contour.

In conclusion we note, that inclusion of the higher order derivatives in the defining parameters results in still higher values of m.

The author thanks L. I. Sedov for assessing this paper and V. A. Zhelnorovich for his comments.

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Translated by L. K.